

Numerical method of inertance tube pulse tube refrigerator [☆]

Shaowei Zhu ^{a,*}, Yoichi Matsubara ^b

^a *Second Development Department, Aisin Seiki Co. Ltd., 2-1, Asahi-machi, Kariya, Aichi 448-8650, Japan*

^b *Institute of Quantum Science, Nihon University, 7-24-1, Narashinodai, Funabashi, Chiba 274-8501, Japan*

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Abstract

A nodal analysis method for simulating inertance tube pulse tube refrigerators is introduced. The energy equation, continuity equation, momentum equation of gas, energy equation of solid are included in this model. Boundary condition can be easily changed to enable the numerical program calculate thermal acoustic engines, inertance tube pulse tube refrigerators, double inlet pulse tube refrigerators, and others. Implicit control volume method is used to solve these equations. In order to increase the calculation speed, the continuity equation is changed to pressure equation with ideal gas assumption, and merged with momentum equation. Then the algebraic equation group from continuity and momentum equation becomes one group. With this numerical method, an example calculation of a large scale inertance tube pulse tube refrigerator is shown.

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1. Introduction

An inertance tube pulse tube refrigerator is a very important type pulse tube refrigerator [1]. There are several parameters in this machine. Numerical simulation is an effective way to explain the working mechanism and also an effective way to find the rough optimum point for the initial design [2]. In this paper, a nodal analysis method based on one-dimensional compressible gas flow model is introduced. The energy equation, continuity equation, momentum equation of gas and energy equation of solid are included. Acceleration method is used for increasing the calculation speed.

Thermal acoustic engine [3] is a no moving parts compressor for pulse tube refrigerator. The development of pulse tube refrigerator without moving parts is also very important. This method is also designed for calculation of thermal acoustic engine [4] because thermal acoustic engine and inertance pulse tube refrigerator are

essentially the same from the numerical viewpoint. It is also designed as a general method that can be easily changed to simulate other type of refrigerators, such as double inlet pulse tube refrigerator, orifice pulse tube refrigerator, GM refrigerator, and Stirling refrigerator. With this numerical method, a large scale inertance tube pulse tube refrigerator is taken as an example calculation.

2. Inertance tube pulse tube refrigerator and acoustic engine

Fig. 1 is the schematic of an inertance tube pulse tube refrigerator. The volume of compressor 1 changes sinusoidal to cause the pressure oscillation within the cooler 2, regenerator 3, cold heat exchanger 4 and pulse tube 5. The gas in the inertance tube also oscillates.

Fig. 2 shows an acoustic engine. Heat is supplied to the hot heat exchanger 2, and part of the heat is changed to work by stack 3, the waste heat is discharged from the cold heat exchanger 4.

From mathematical viewpoint, the above two machines are the same, only boundary condition is different. So the same numerical model is applicable.

[☆] An calculation example is at <http://homepage3.nifty.com/cryocooler/>.

* Corresponding author.

E-mail addresses: swzhu@rd.aisin.co.jp (S. Zhu), cryo@nifty.com (Y. Matsubara).

Nomenclature

A	cross gas flow area
A_s	cross solid area
A_L	heat transfer area per meter
c	sound velocity
C_V	constant volume specific heat of gas
C_P	constant pressure specific heat of gas
C_S	specific heat of solid
D	diameter of inertance tube
D_e	hydrodiameter
E	exergy flow
f	friction coefficient
H	enthalpy flow
H_{PT}	pulse tube enthalpy flow
H_{reg}	regenerator loss
L	length of inertance tube
L_1	the distance of the lowest pressure ratio point to the left end of the inertance tube
L_2	the distance of the lowest pressure ratio point to the right end of the inertance tube
$L1$	the last nodal number
$L2$	the nodal number in the last control volume
Lbs	the first nodal in the left end of the right buffer
LPE	the last nodal of pulse tube
LPS	the first nodal of pulse tube
m	mass of gas
m_s	mass of solid
$\dot{m}$	mass flow rate
P	pressure
Q	cooling power
R	gas constant
s	entropy
S	entropy flow

t	time
T	temperature of gas
T_0	room temperature
T_S	temperature of matrix
u	velocity
V	volume
V_2	volume of compressor
V_{2S}	dead volume of compressor
V_{20}	swept volume of compressor
V_b	buffer volume
V_p	pulse tube volume
V_{tip1}	tip1 volume
V_{tip2}	tip2 volume
W	compressor work
x	coordinator along the length

Greek letters

δX	X step length
δt	time step length
α	heat transfer coefficient
β	coefficient for equivalent pulse tube volume
ρ	density of gas
ρ_S	density of matrix
τ	period
ω	angle frequency

Subscripts

f	interface of each control volume
i	number of control volume
cv	center of main control volume

Superscript

k	time step
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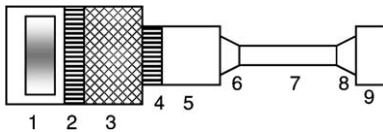


Fig. 1. Inertance tube pulse tube refrigerator. 1. Compressor, 2. cooler, 3. regenerator, 4. cold heat exchanger, 5. pulse tube, 6. tip1, 7. inertance tube, 8. tip2 and 9. buffer.

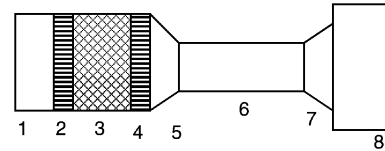


Fig. 2. Thermal acoustic engine. 1. Hot buffer, 2. heater, 3. stack, 4. cooler, 5. tip1, 6. inertance tube, 7. tip2 and 8. buffer.

3. Numerical model

One-dimensional compressible fluid flow model is introduced. The control equations are following [5].

Continuity equation

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0 \quad (1-1)$$

Momentum equation

$$\frac{\partial}{\partial t}(\rho A u) + \frac{\partial}{\partial x}(\rho A u^2) + A \frac{\partial P}{\partial x} + A \frac{f}{D_e} \frac{1}{2} \rho u^2 \frac{u}{|u|} = 0 \quad (1-2)$$

Energy equation for gas

$$\frac{\partial}{\partial t} \left[\rho A \left(C_v T + \frac{u^2}{2} \right) \right] + \frac{\partial}{\partial x} \left[\rho u A \left(C_p T + \frac{u^2}{2} \right) \right] + \alpha A_L (T - T_s) = 0 \quad (1-3)$$

Energy equation for matrix

$$A_S \rho_S C_S \frac{\partial T_s}{\partial t} = \alpha A_L (T - T_s) \quad (1-4)$$

4. Numerical method

Finite difference method is used to solve the above equations [6–8]. Each component such as pulse tube, regenerator is divided into small control volumes. All of the control volumes are put together, then we get the mesh system as shown in Fig. 3. Each box means one control volume. The black point means the nodal which is in the center of the control volume. The temperature, mass, density, pressure, and gas property are the values at the center of each control volume. The mass flow rate and velocity are the values at the interface whose number is the number of right side nodal number. The volume between each dotted line is the control volume for mass flow rate and velocity. The control volume 2 and L2 are changeable in order to change the program easily for simulation Stirling refrigerators, GM refrigerators, and others.

In time coordinate, the first order method is used at the first step. After step one, the second order method is used. In x coordinate, the second order up-wind method [9] is used mainly. To avoid the numerical noise, at both ends of each component, first order up-wind method is used.

The continuity equation and momentum equation are combined. First we change the continuity equation to the form $P_i = f(\dot{m}_i, \dot{m}_{i+1})$ under the assumption of ideal gas, and combine it with the momentum equation. The momentum equation and continuity equation are changed to one algebraic equation groups. Compare to the method in Ref. [6,7], the algebraic equations groups which we have to use the iteration method to solve is decreased to one group from two groups. The calculation speed is increased.

4.1. Continuity equation

Eq. (1-1) is changed to the following form with the first order method

$$\frac{m_i - m_i^{k-1}}{\delta t} = \dot{m}_i - \dot{m}_{i+1} \quad (2-1)$$

Eq. (2-1) can be changed to the pressure equation with the ideal gas state equation by the first order method.

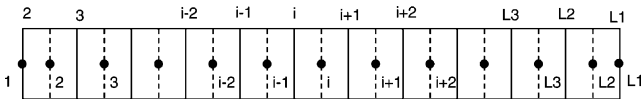


Fig. 3. Grid and control volume of regenerator.

$$\frac{V_i P_i}{RT_i} = m_i = (\dot{m}_i - \dot{m}_{i+1})\delta t + m_i^{k-1} \quad (2-2)$$

$$P_i = \left[(\dot{m}_i - \dot{m}_{i+1})\delta t + m_i^{k-1} \right] \frac{RT_i}{V_i} \quad (2-3)$$

$$P_i = (\dot{m}_i - \dot{m}_{i+1})a_i + b_i \quad (2-4)$$

$$a_i = \delta t \frac{RT_i}{V_i} \quad (2-5)$$

$$b_i = m_i^{k-1} \frac{RT_i}{V_i} \quad (2-6)$$

Eq. (1-1) can be changed to the pressure equation by the second order method

$$\frac{1.5m_i - 2m_i^{k-1} + 0.5m_i^{k-2}}{\delta t} = \dot{m}_i - \dot{m}_{i+1} \quad (2-7)$$

$$\frac{1.5V_i P_i}{RT_i} = 1.5m_i = (\dot{m}_i - \dot{m}_{i+1})\delta t + 2m_i^{k-1} - 0.5m_i^{k-2} \quad (2-8)$$

$$P_i = \left[(\dot{m}_i - \dot{m}_{i+1})\delta t + 2m_i^{k-1} - 0.5m_i^{k-2} \right] \frac{RT_i}{1.5V_i} \quad (2-9)$$

$$P_i = (\dot{m}_i - \dot{m}_{i+1})a_i + b_i \quad (2-10)$$

$$a_i = \delta t \frac{RT_i}{1.5V_i} \quad (2-11)$$

$$b_i = (2.0m_i^{k-1} - 0.5m_i^{k-2}) \frac{RT_i}{1.5V_i} \quad (2-12)$$

4.2. Momentum equation

Eq. (1-2) can be changed to

$$\frac{\partial}{\partial t}(\dot{m}) + \frac{\partial}{\partial x}(\dot{m}u) + A \frac{\partial P}{\partial x} + \frac{f}{D} \frac{1}{2} \dot{m}|u| = 0 \quad (3-1)$$

$$\begin{aligned} \frac{\partial \dot{m}_i}{\partial t} \delta x_i + (\dot{m}u)_{\text{cvi}} - (\dot{m}u)_{\text{cvi-1}} + A_i(P_i - P_{i-1}) \\ + \frac{\delta x_i}{D_{ei}} f_i \frac{1}{2} \dot{m}_i |u_i| = 0 \end{aligned} \quad (3-2)$$

$(\dot{m}u)_{\text{cvi}}$ and $(\dot{m}u)_{\text{cvi-1}}$ is on the nodal point.

Each term in Eq. (3-2) can be changed to the following term.

By the first order method

$$\frac{\partial \dot{m}_i}{\partial t} \delta x_i = \frac{\dot{m}_i - \dot{m}_i^{k-1}}{\delta t} \delta x_i \quad (3-3)$$

By the second order method

$$\frac{\partial \dot{m}_i}{\partial t} \delta x_i = \frac{1.5\dot{m}_i - 2\dot{m}_i^{k-1} + 0.5\dot{m}_i^{k-2}}{\delta t} \delta x_i \quad (3-4)$$

By Eq. (2-4) or (2-10)

$$A_i(P_i - P_{i-1}) = A_i a_i \dot{m}_i - A_i a_i \dot{m}_{i+1} + A_i b_i - A_i a_{i-1} \dot{m}_{i-1} + A_i a_{i-1} \dot{m}_i - A_i b_{i-1} \quad (3-5)$$

The term $(\dot{m}u)_{\text{cvi}} - (\dot{m}u)_{\text{cvi}-1}$ is a little complex. The second orders up-wind method has been used except both ends of each component.

$$\dot{m}_{\text{cvi}} = \begin{cases} a_{1i} \dot{m}_i - b_{1i} \dot{m}_{i-1} & u_{\text{cvi}} > 0 \\ a_{2i} \dot{m}_{i+1} - b_{2i} \dot{m}_{i+2} & u_{\text{cvi}} < 0 \end{cases} \quad (3-6)$$

$$\dot{m}_{\text{cvi}-1} = \begin{cases} a_{1i-1} \dot{m}_{i-1} - b_{1i-1} \dot{m}_{i-2} & u_{\text{cvi}-1} > 0 \\ a_{2i-1} \dot{m}_i - b_{2i-1} \dot{m}_{i+1} & u_{\text{cvi}-1} < 0 \end{cases} \quad (3-7)$$

$a_{1i} = 1.5$, $b_{1i} = 0.5$, $a_{2i} = 1.5$, $b_{2i} = 0.5$ for second order method, $a_{1i} = 1$, $b_{1i} = 0$, $a_{2i} = 1$, $b_{2i} = 0$ for first order method.

The velocity is changed to the following form

$$A_{1i} = \begin{cases} u_{\text{cvi}} & u_{\text{cvi}} > 0 \\ 0 & u_{\text{cvi}} < 0 \end{cases} \quad (3-8)$$

$$B_{1i} = \begin{cases} 0 & u_{\text{cvi}} > 0 \\ u_{\text{cvi}} & u_{\text{cvi}} < 0 \end{cases} \quad (3-9)$$

Then

$$(\dot{m}u)_{\text{cvi}} - (\dot{m}u)_{\text{cvi}-1} = A_{1i}(a_{1i} \dot{m}_i - b_{1i} \dot{m}_{i-1}) + B_{1i}(a_{2i} \dot{m}_{i+1} - b_{2i} \dot{m}_{i+2}) - A_{1i-1}(a_{1i-1} \dot{m}_{i-1} - b_{1i-1} \dot{m}_{i-2}) - B_{1i-1}(a_{2i-1} \dot{m}_i - b_{2i-1} \dot{m}_{i+1}) \quad (3-10)$$

If the flow is laminar flow with the friction equation

$$f = \frac{C_{\text{Re}}}{Re} \quad (3-11)$$

Then

$$\frac{\delta x_i}{D_{\text{ei}}} f_i \frac{1}{2} \dot{m}_i |u_i| = \frac{1}{2} \frac{\mu_{\text{wi}} C_{\text{Re}} \delta x_i}{D_{\text{ei}}^2 \rho_{\text{wi}}} \dot{m}_i \quad (3-12)$$

If the flow is turbulent flow, the friction equation can be getting from Ref. [10].

Then the momentum equation can be changed to

$$a_{\text{pi}} \dot{m}_i = a_{\text{wi}} \dot{m}_{i-1} + a_{\text{ci}} \dot{m}_{i+1} + b_{\text{pi}} \quad (3-13)$$

The momentum equation is not considered in volume 2 and volume L2.

4.3. Energy equation of gas

Energy equation can be changed to equation

$$\frac{\partial C_{\text{vi}} \dot{m}_i T_i}{\partial t} + C_{\text{pi}+1} \dot{m}_{i+1} T_{i+1} - C_{\text{pi}} \dot{m}_i T_i + \alpha_i A_{\text{Li}} (T_i - T_{\text{Si}}) + 0.5 \left(\frac{\partial \dot{m}_i u_{\text{cvi}}^2}{\partial t} + \dot{m}_{i+1} u_{i+1}^2 - \dot{m}_i u_i^2 \right) = 0 \quad (4-1)$$

By the first order method in time direction

$$\frac{\partial C_{\text{vi}} \dot{m}_i T_i}{\partial t} = \frac{C_{\text{vi}} \dot{m}_i T_i - C_{\text{vi}}^{k-1} \dot{m}_i^{k-1} T_i^{k-1}}{\delta t} \quad (4-2)$$

By the second order method in time direction

$$\frac{\partial C_{\text{vi}} \dot{m}_i T_i}{\partial t} = \frac{1.5 C_{\text{vi}} \dot{m}_i T_i - 2.0 C_{\text{vi}}^{k-1} \dot{m}_i^{k-1} T_i^{k-1} + 0.5 C_{\text{vi}}^{k-2} \dot{m}_i^{k-2} T_i^{k-2}}{\delta t} \quad (4-3)$$

In x direction, the second order up-wind method are used except both ends.

$$T_{\text{fi}} = \begin{cases} a_{3i} T_{i-1} - b_{3i} T_{i-2} & \dot{m}_i > 0 \\ a_{4i} T_i - b_{4i} T_{i+1} & \dot{m}_i < 0 \end{cases} \quad (4-4)$$

$$T_{\text{fi}+1} = \begin{cases} a_{3i+1} T_i - b_{3i+1} T_{i-1} & \dot{m}_{i+1} > 0 \\ a_{4i+1} T_{i+1} - b_{4i+1} T_{i+2} & \dot{m}_{i+1} < 0 \end{cases} \quad (4-5)$$

$a_{3i} = 1.5$, $b_{3i} = 0.5$, $a_{4i} = 1.5$, $b_{4i} = 0.5$ for the second order method, $a_{3i} = 1$, $b_{3i} = 0$, $a_{4i} = 1$, $b_{4i} = 0$ for the first order method. To avoid numerical noise, at both end of each component such as regenerator, or the inertance tube, the first order up-wind method and the second order up-wind method are mixed. Fig. 4 shows how to use the first order up-wind method and the second order up-wind method near the two components such as the regenerator and the cooler. Fig. 5 shows how to use the first order up-wind method and second order up-wind method at both end of the machine. In Figs. 4 and 5, the arrow to right means gas flow to right direction, the arrow to left means the gas flows to left direction. The 2nd means the second order up-wind, the 1st means the first order up-wind method.

The mass flow rate timing heat capacity is changed to the following

$$A_{2i} = \begin{cases} C_{\text{pi}} \dot{m}_i & \dot{m}_i > 0 \\ 0 & \dot{m}_i < 0 \end{cases} \quad (4-6)$$

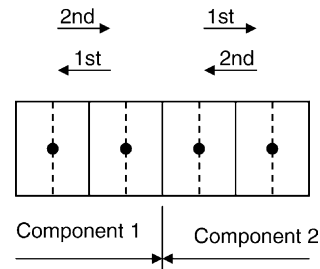


Fig. 4. First order up-wind method near the interface of two components.

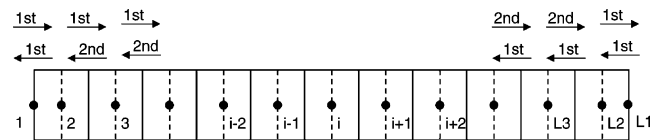


Fig. 5. First order up-wind method at both end of the machine.

$$B_{2i} = \begin{cases} 0 & \dot{m}_i > 0 \\ C_{pi}\dot{m}_i & \dot{m}_i < 0 \end{cases} \quad (4-7)$$

Then

$$\begin{aligned} C_{pi+1}\dot{m}_{i+1}T_{i+1} - C_{pi}\dot{m}_iT_{fi} &= A_{2i+1}(a_{3i+1}T_i - b_{3i+1}T_{i-1}) \\ &+ B_{2i+1}(a_{4i+1}T_{i+1} - b_{4i+1}T_{i+2}) \\ &- A_{2i}(a_{3i}T_{i-1} - b_{3i}T_{i-2}) \\ &- B_{2i}(a_{4i}T_i - b_{4i}T_{i+1}) \end{aligned} \quad (4-8)$$

The final equation of the energy equations of gas and matrix are changed to

$$a_{pi}T_i = a_{ei}T_{i-1} + a_{wi}T_{i+1} + b_{pi} \quad (4-9)$$

4.4. Energy equation of solid

Matrix equation (1-4) also can be changed to

$$\frac{\partial C_{Si}T_{Si}}{\partial t} + \alpha_i A_{Li}(T_{Si} - T_i) = 0 \quad (5-1)$$

By the first order method

$$\frac{\partial C_{Si}T_{Si}}{\partial t} = \frac{C_{Si}T_{Si} - C_{Si}^{k-1}T_{Si}^{k-1}}{\delta t} \quad (5-2)$$

By the second order method

$$\frac{\partial C_{Si}T_{Si}}{\partial t} = \frac{1.5C_{Si}T_{Si} - 2.0C_{Si}^{k-1}T_{Si}^{k-1} + 0.5C_{Si}^{k-2}T_{Si}^{k-2}}{\delta t} \quad (5-3)$$

Finally we get

$$a_{pi}T_{Si} = b_{pi} \quad (5-4)$$

4.5. Buffer

There is alternate of the dealing with the momentum equation in the right buffer. The momentum effect in the buffer is almost zero. We can consider the pressure in the buffer is the same and the velocity is zero on each time step. It is not necessary to solve the momentum equations in the buffer. The calculation speed will be increased.

Apply Eq. (2-2) in the buffer from $i = Lbs$ to $L2$, then we get the pressure in the buffer with the first order method.

$$P_{Lbs} = \frac{(\dot{m}_{Lbs} - \dot{m}_{L1})\delta t + \sum_{i=Lbs}^{L2} m_i^{k-1}}{\sum_{i=Lbs}^{L2} \frac{V_i}{RT_i}} \quad (6-1)$$

Apply Eq. (2-8) in the buffer from $i = Lbs$ to $L2$, then we get the pressure in the buffer with the second order method.

$$P_{Lbs} = \frac{(\dot{m}_{Lbs} - \dot{m}_{L1})\delta t + 2 \sum_{i=Lbs}^{L2} m_i^{k-1} - 0.5 \sum_{i=Lbs}^{L2} m_i^{k-2}}{1.5 \sum_{i=Lbs}^{L2} \frac{V_i}{RT_i}} \quad (6-2)$$

We can use the same method to deal with the left buffer.

4.6. Boundary

The volume of the compressor

$$V_2 = V_{2S} + 0.5V_{20}(1 - \sin(\omega t)) \quad (7-1)$$

V_2 is a part of buffer if it is an acoustic engine.

The volume of $L2$ also can be changeable. The pressure drop equation and heat transfer equation are taken from reference [10]. The wall temperatures of the heat exchangers, buffers and inertance tube are constant.

4.7. Total parameters

Enthalpy flow

$$H = \frac{1}{\tau} \oint \dot{m} \left(C_p + \frac{1}{2}u^2 \right) dt \quad (8-1)$$

Entropy flow

$$S = \frac{1}{\tau} \oint \dot{m} s dt \quad (8-2)$$

Exergy flow

$$E = H - ST_0 \quad (8-3)$$

Compressor input work

$$W = \frac{1}{\tau} \oint P_2 dV_2 \quad (8-4)$$

$$Q = H_{PT} - H_{reg} \quad (8-5)$$

$$COP = Q/W \quad (8-6)$$

Equivalent PV diagrams are given from the both ends of the gas piston within the pulse tube. The gas piston is defined as follows. The maximum left position of the left end of the gas piston just touch the left end of the pulse tube. The maximum right position of the right end of the gas piston just touch the right end of the pulse tube. The following term is introduced for getting the position of the left and right end of the gas position.

$$M_i^k = m_{right}^k + \sum_{i=i}^{LPE} m_i^k \quad (8-7)$$

where m_{right} is the gas within the imaginary cylinder at the hot end of the pulse tube.

$$m_{right}^k = m_{right0} + \sum_{k=1}^k \dot{m}_{LPE+1}^k \quad (8-8)$$

m_{right0} is chosen to let the minimum of m_{right}^k become zero.

The left end of the gas piston is represented by

$$M_{LPS} = \min (M_{LPS}^k) \quad (8-9)$$

The right end of the gas piston is represent by

$$M_{LPE} = \max (M_{LPE+1}^k) \quad (8-10)$$

If $M_i^k < M_{LPS} < M_{i+1}^k$, we can know the left end position of the gas piston

$$X_{LPS}^k = x_i + \frac{x_i - x_{i+1}}{M_i^k - M_{i+1}^k} (M_i^k - M_{LPS}) \quad (8-11)$$

If $M_i^k < M_{LPE} < M_{i+1}^k$, We also can know the right end position of the gas piston

$$X_{LPE}^k = x_i + \frac{x_i - x_{i+1}}{M_i^k - M_{i+1}^k} (M_i^k - M_{LPE}) \quad (8-12)$$

4.8. The solve process

The present program cannot let the gas oscillate by itself in acoustic engine. An initial small velocity is given in the program. The solving processes are as follows:

- (1) Temperature field
- (2) Mass flow rate field
- (3) Pressure field

It is repeated from (1) to (3) until one time layer is finished, and then goes to next time layer till one period is finished. The calculation is continued till the required calculation accuracy that is judged by how near of the enthalpy flow rate at both end of regenerator and pulse tube. At each iteration step, tri-diagonal matrix algorithm (TDMA) method is used for solving Eqs. (3-13) and (4-9).

For acoustic engine, during the calculation, the maximum pressure point in the hot buffer is checked out to get the oscillation period, which is given from two maximum pressure points.

5. Other regenerator type refrigerators

The present simulation program is also applicable for other regenerator type refrigerators, such as Stirling type pulse tube refrigerators, GM type pulse tube refrigerators, Stirling refrigerators, GM refrigerators, VM refrigerators, and so on. The difference is boundary. The momentum term is not necessary in this kind of refrigerators.

6. Calculation example of large scale inertance tube pulse tube refrigerator

Here the big pulse tube refrigerator is taken as an example whose regenerator with diameter 110 mm and length 50 mm and pulse tube with diameter 38 mm and length 150 mm is about 20 times large than those in Ref. [11]. In Ref. [11], the cooling power is less than 20 W, but the cooling power is hundred watts in the present refrigerator. Thus we call this pulse tube refrigerator as a large scale inertance tube pulse tube refrigerator. Room temperature is 300 K. Refrigeration temperature is 80 K. Working medium is helium gas. Average pressure is 2 MPa. Working frequency is 50 Hz. The wall temperature of the compressor, the cooler, the tips, the inertance tube, and the buffer is room temperature. The wall temperature of the cold heat exchanger is refrigeration temperature. Regenerator matrix and pulse tube wall only exchanges heat with helium gas. The expansion work of the pulse tube is very large in the big inertance tube pulse tube refrigerator. The gas in the inertance tube can strongly oscillate due to the big work input to the inertance tube.

6.1. Optimum point

By Ref. [2], if other parameters are fixed, there is an optimum diameter and length of the inertance tube. Fig. 6a–d shows the inertance tube length dependence of the input power of the compressor, cooling power, pressure ratio in the pulse tube, and COP, for different diameter for the inertance tube when the buffer volume is 4 l and the compressor swept volume is 150 cc. The peak values of the cooling power, pressure ratio in the pulse tube and COP become the largest with the inertance tube with 16 mm in diameter. The input power with inertance tube of 16 mm in diameter is only slightly lower than that with inertance tube of 20 mm in diameter. So the inertance tube of 16 mm in diameter is considered as optimum. With inertance tube of 16 mm in diameter, the COP gets peak of 0.165 with length 3.75 m, and the peak cooling power over 600 W and peak input power about 4000 W gets with length 3.375 m.

With inertance tube in diameter 16 mm and length 3.75 m, the equivalent PV diagrams within the pulse tube are shown in Fig. 7, the mass flow rate and pressure wave at the cold end of the pulse tube and the center of the regenerator are shown in Fig. 8. Compared to the small pulse tube refrigerator in Ref. [2], a large phase lead of the pressure wave is achieved relative to the mass flow rate at the cold end of the pulse tube, and the PV diagram at the cold end is tilted to the left largely. The pressure wave and mass flow rate are almost in phase at the middle of the regenerator. So on the optimum point of large scale inertance tube pulse tube refrigerator, the PV diagram at the cold end of the pulse tube is tilted to

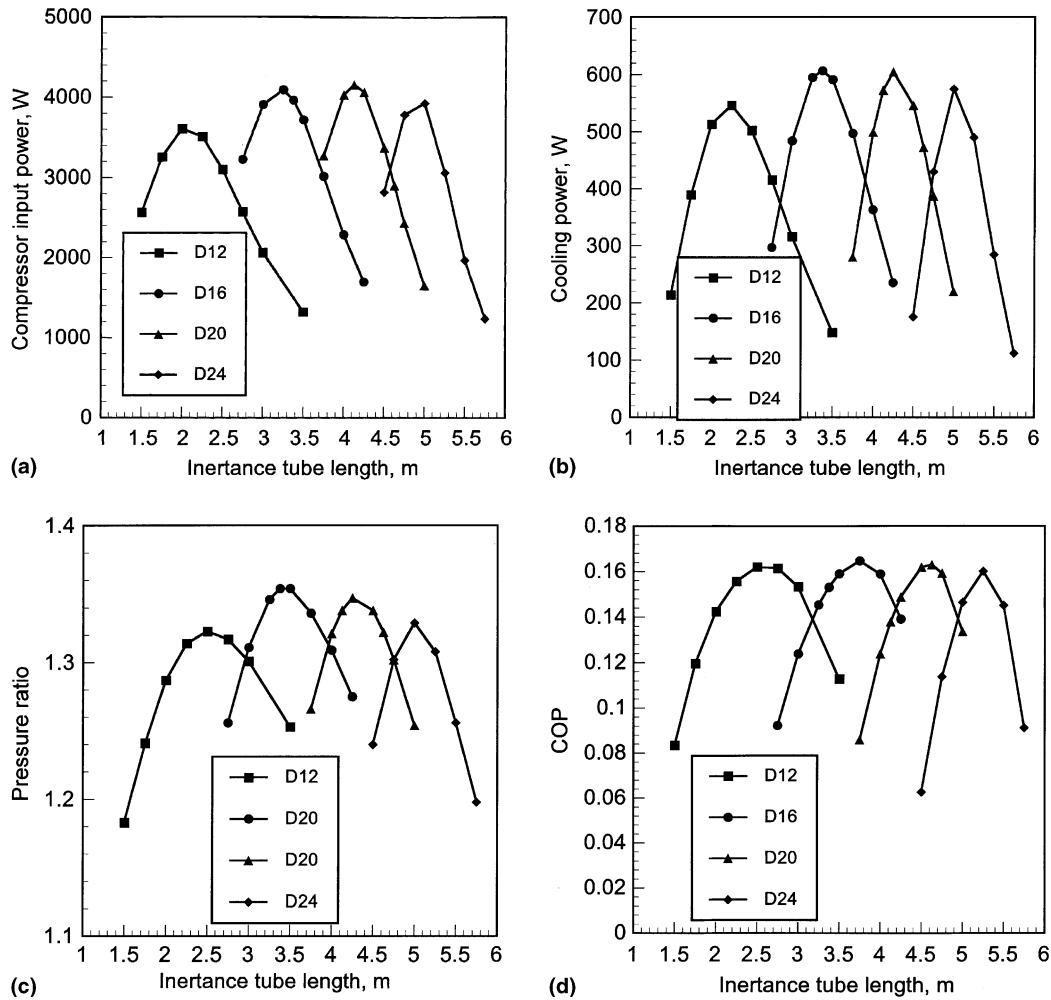


Fig. 6. (a) Compressor input work: D12 inertance tube diameter 12 mm, D16 inertance tube diameter 16 mm, D20 inertance tube diameter 12 mm, D24 inertance tube diameter 16 mm. (b) Cooling power: D12 inertance tube diameter 12 mm, D16 inertance tube diameter 16 mm, D20 inertance tube diameter 12 mm, D24 inertance tube diameter 16 mm. (c) Pressure ratio in pulse tube: D12 inertance tube diameter 12 mm, D16 inertance tube diameter 16 mm, D20 inertance tube diameter 12 mm, D24 inertance tube diameter 16 mm. (d) COP: D12 inertance tube diameter 12 mm, D16 inertance tube diameter 16 mm, D20 inertance tube diameter 12 mm, D24 inertance tube diameter 16 mm.

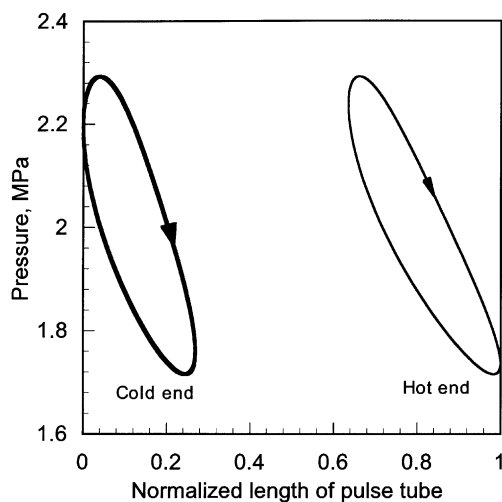


Fig. 7. Equivalent PV diagrams in pulse tube.

the left, the pressure wave leads a large phase to mass flow rate at the cold end of the pulse tube, and mass flow rate and pressure wave in phase point is inside of the regenerator.

6.2. Inertance tube

The inertance tube is an important part in the inertance tube pulse tube refrigerator. The pressure wave, mass flow rate and velocity in the inertance tube is useful for understanding how the inertance tube working in large scale pulse tube refrigerator.

With inertance tube in diameter 16 mm and length 3.75 m, the maximum and minimum pressure and velocity in the inertance tube are shown in Fig. 9. Here X is normalized by the total length of the inertance tube directed from left to right. Also, the mass flow rate and pressure at $X = 0, 0.5, 0.87$, and 1.0 are shown in

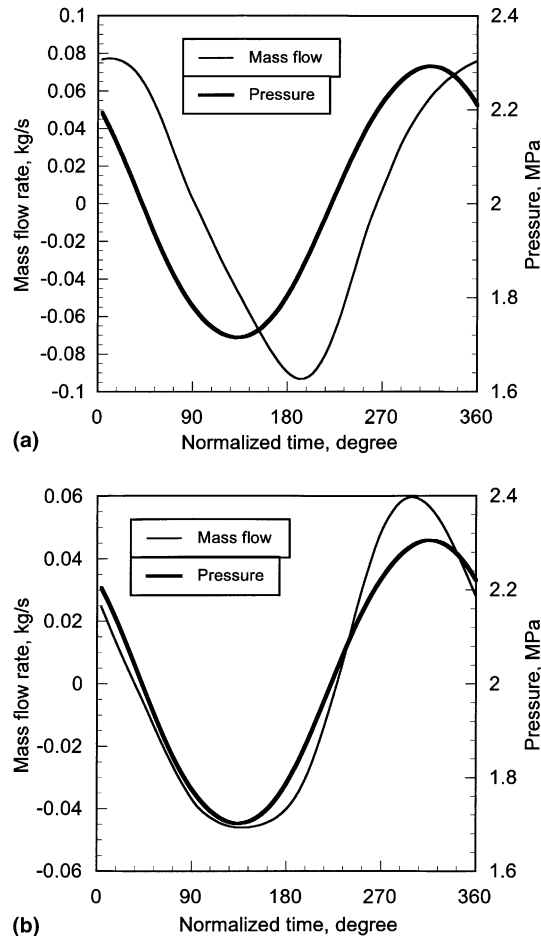


Fig. 8. (a) Pressure wave and mass flow rate at pulse tube cold end and (b) pressure wave and mass flow rate at regenerator center.

Fig. 10a and b. As shown in Fig. 9, the pressure amplitude decreases from the left end to the lowest pressure point ($X = 0.87$) where the pressure amplitude is the smallest, and increases again, while the velocity shows a maximum near the right. Fig. 10a shows that the pressure amplitude decreases from the left to the center while the phase does not vary much. At $X = 0.87$ frequency is twice as large as the base frequency because the amplitude at the second harmonics is larger than that at the base frequency. At the right end, the phase of the pressure wave is changed by 180° from the others. Fig. 10b shows that the phase of the mass flow rate is kept almost constant throughout the inductance tube. In most part of the inductance tube, the phase angle between the mass flow rate and pressure wave is about 90° . So when the velocity is at the maximum, the pressure is almost at zero amplitude, when the pressure is at the maximum amplitude, the velocity is close to zero. Fig. 11a and b show the instantaneous pressure and velocity distribution in the inductance tube, respectively. Indicated degree correspond to the instance of a cycle which is divided into 360° . It is shown that the pressure is almost linear distribution in the inductance tube in any time. The

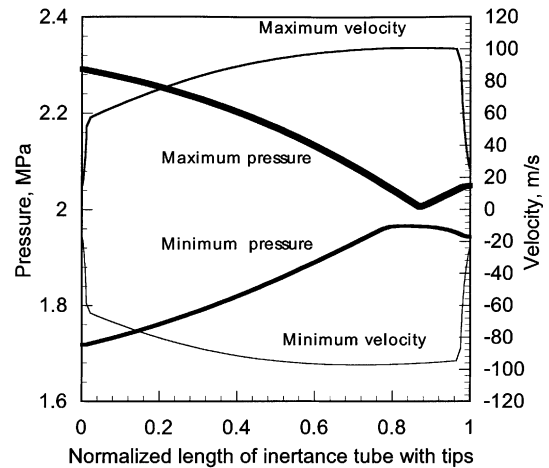


Fig. 9. Maximum and minimum pressure and velocity in inductance tube.

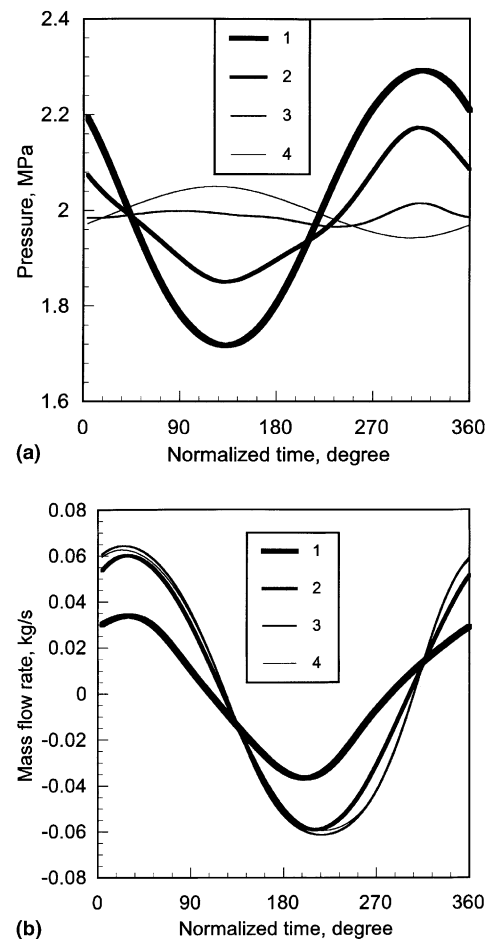


Fig. 10. (a) Pressure wave in inductance tube: 1. hot end of pulse tube, 2. center of inductance tube, 3. lowest pressure ratio point and 4. buffer. (b) Mass flow rate in inductance tube: 1. hot end of pulse tube, 2. center of inductance tube, 3. lowest pressure ratio point and 4. buffer.

peak velocity is about 100 m/s. So large inductance effect could be obtained.

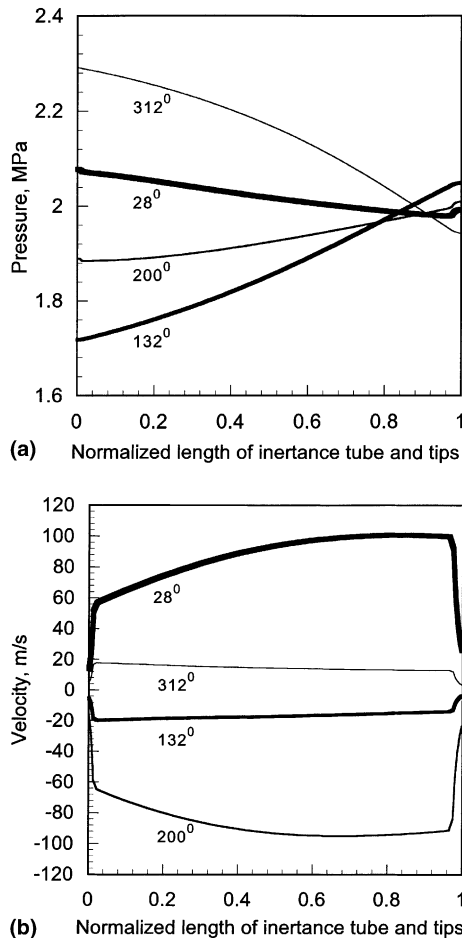


Fig. 11. (a) Pressure distribution in inertance tube, (b) velocity distribution in inertance tube.

When the inertance tube of 16 mm in diameter and 3.75 m in length, the exergy flow and enthalpy flow along the refrigerator are shown in Fig. 12. In Fig. 12, coordinate X for the different components in the refrigerator is normalized by their respective length because the length of the regenerator and pulse tube is millimeter order while the length of the inertance tube is meter order. The range $0 < X < 1$ represents the cooler and the regenerator; the range $1 < X < 2$ represents the cold heat exchanger and the pulse tube; the range $2 < X < 3$ represents the tip1, inertance tube tip2, and buffer. The exergy flow near 3000 W is inputted from the compressor. It decreases to about 2000 W at the cold end of the regenerator. About 1500 W of it is taken out as cooling power through the cold head exchanger. Finally, the exergy flow over 500 W flows from the left end of the pulse tube into the inertance tube and gradually decreases to zero at the end of the inertance tube. The over 500 W exergy flow is the expansion work of the gas in the cold end of the pulse tube. It is the power for the gas oscillation in the inertance tube. Fig. 12 also shows that the loss in the regenerator is very large even in the large scale inertance tube pulse tube refrigerator.

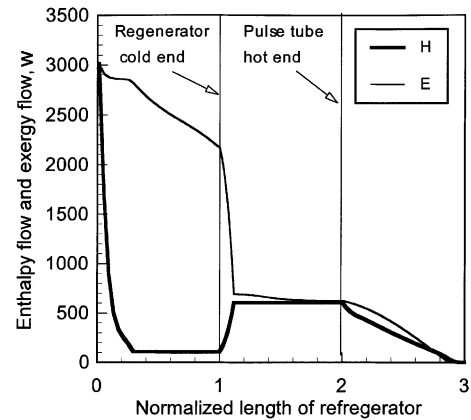


Fig. 12. Exergy flow and enthalpy flow.

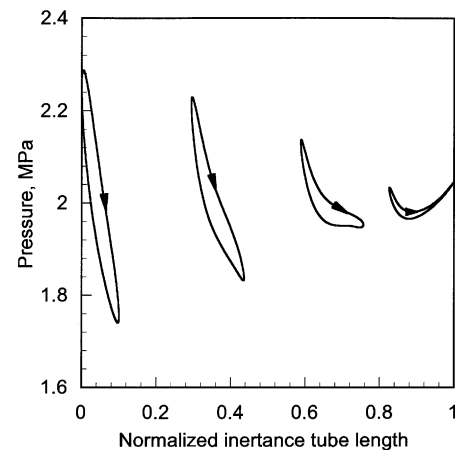


Fig. 13. Equivalent PV diagrams in inertance tube.

Similar to the pulse tube, we also can get equivalent PV diagrams in the inertance tube as shown in Fig. 13 for the gas pistons dividing the inertance tube into three. From left end to right end, the size of the PV diagrams decreases, the shape is also changed.

The numerical results also show that the distance from the lowest pressure ratio point to the buffer almost does not change regardless of the length of the inertance tube, but it increases with the increasing of the diameter of the inertance tube when the buffer volume is 4 l. In other words, if the diameter of the inertance tube and the buffer volume is fixed, the distance from the lowest pressure ratio point to the hot end of the pulse tube is changed if we change the length of inertance tube.

6.3. Buffer size and compressor size effect

With the inertance tube of 16 mm in diameter, the input power of the compressor, cooling power, pressure ratio in the pulse tube, and COP are shown in Fig. 14 for different buffer sizes (1, 2, 4, 10 l) and swept volume (150, 200, 250 cc) of the compressor. With decreasing

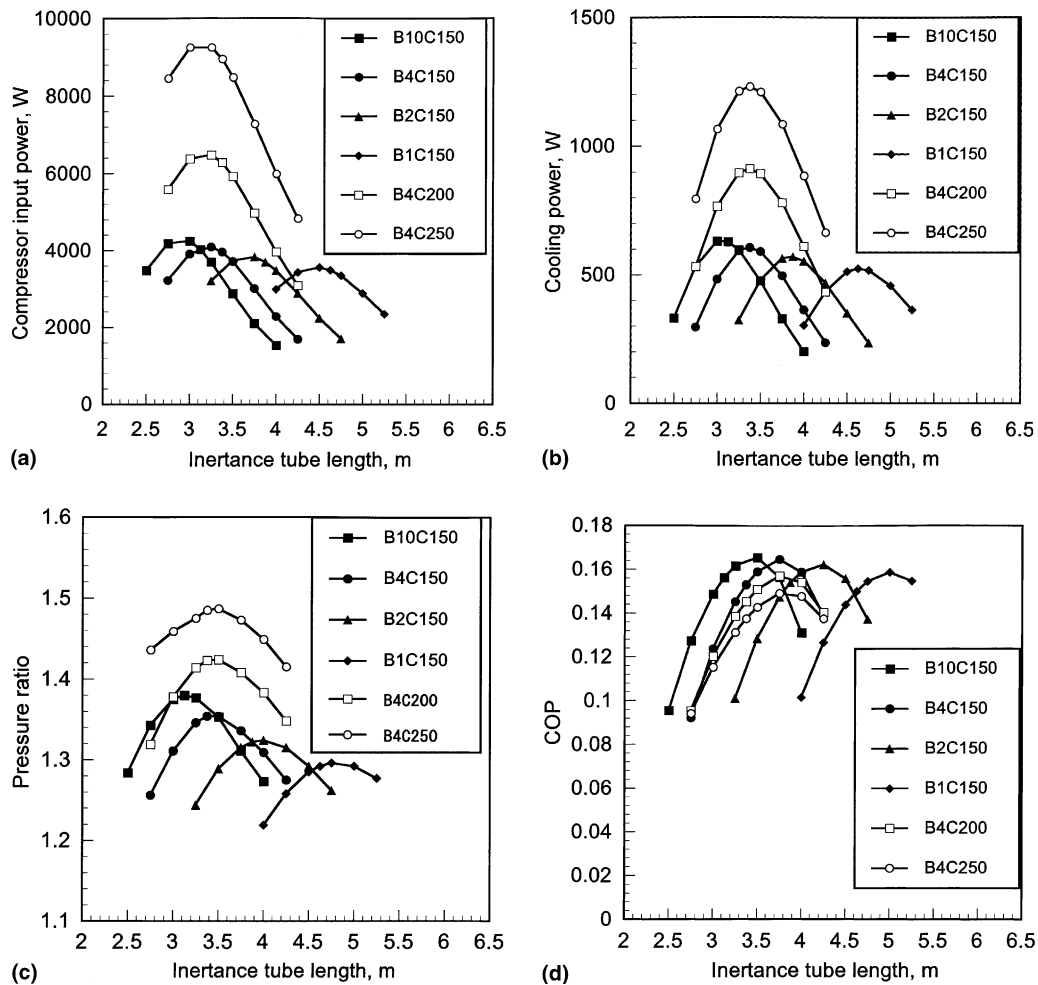


Fig. 14. (a) Buffer volume and compressor swept volume effect to compressor input work: B10C150 buffer volume 10 l and compressor swept volume 150 cc, B4C150 buffer volume 4 l and compressor swept volume 150 cc, B2C150 buffer volume 2 l and compressor swept volume 150 cc, B1C150 buffer volume 1 l and compressor swept volume 150 cc, B4C200 buffer volume 4 l and compressor swept volume 200 cc, B4C250 Buffer volume 4 l and compressor swept volume 250 cc. (b) Buffer volume and compressor swept volume effect to cooling power. B10C150 buffer volume 10 l and compressor swept volume 150 cc, B4C150 buffer volume 4 l and compressor swept volume 150 cc, B2C150 buffer volume 2 l and compressor swept volume 150 cc, B1C150 buffer volume 1 l and compressor swept volume 150 cc, B4C200 Buffer volume 4 l and compressor swept volume 200 cc, B4C250 buffer volume 4 l and compressor swept volume 250 cc. (c) Buffer volume and compressor swept volume effect to pressure ratio in pulse tube: B10C150 buffer volume 10 l and compressor swept volume 150 cc, B4C150 buffer volume 4 l and compressor swept volume 150 cc, B2C150 Buffer volume 2 l and compressor swept volume 150 cc, B1C150 buffer volume 1 l and compressor swept volume 150 cc, B4C200 Buffer volume 4 l and compressor swept volume 200 cc, B4C250 buffer volume 4 l and compressor swept volume 250 cc. (d) Buffer volume and compressor swept volume effect to COP. B10C150 Buffer volume 10 l and compressor swept volume 150 cc, B4C150 Buffer volume 4 l and compressor swept volume 150 cc, B2C150 buffer volume 2 l and compressor swept volume 150 cc, B1C150 buffer volume 1 l and compressor swept volume 150 cc, B4C200 Buffer volume 4 l and compressor swept volume 200 cc, B4C250 Buffer volume 4 l and compressor swept volume 250 cc.

the buffer size, the peak values of the input power and cooling power decrease, but the peak COP value remains almost the same. The optimum length of the inertance tube is increased. When the compressor swept volume increases, the input power, cooling power and the pressure ratio in the pulse tube are increase, while the COP decrease, and the optimum length of inertance tube is almost not changed.

With compressor swept volume 150 cc, other numerical results show that when the buffer volume is 10 l, the optimum diameter of inertance tube is 20 mm which is just a little better than the inertance tube of

16 mm in diameter. The optimum diameter of inertance tube is 16 mm in diameter with buffer volume of 1 and 2 l.

With buffer volume of 4 l, other numerical results show that the compressor swept volume almost has no influence on the optimum of inertance tube.

Fig. 15 shows the axial distribution of the pressure ratio in the inertance tube of 16 mm in diameter for different buffer sizes (1, 2, 4, 10 l). The inertance tube length are 5.0, 4.25, 3.75, and 3.5 m, respectively, which are chosen to give the peak COP values for each buffer volume. The swept volume of the compressor is 150 cc.

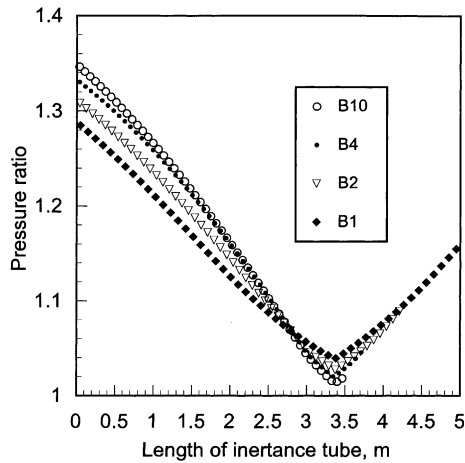


Fig. 15. Pressure ratio in inertance tube. B10 buffer volume 10 l, B4 buffer volume 4 l, B2 buffer volume 2 l, B1 Buffer volume 1.

The distance from the lowest pressure ratio point to its left end is almost not changed with the decrease of the buffer volume, or the lowest pressure point to the right end, is increased in spite of the decrease of the buffer volume.

When the buffer volume is changed, the maximum velocity point in the inertance tube is also changed. The maximum velocity point is near the lowest pressure ratio point which is not shown here.

6.4. Inertance tube and Helmholtz resonator

The working mechanism can be explained more clearly with Helmholtz resonator [12]. In inertance tube pulse tube refrigerator, it can be considered there are two helmholtz resonators. As shown in Fig. 16, the right part of line 0–0 is a helmholtz resonator with the L_2 length inertance tube as the resonator. The part between line 1–1 and 0–0 is another helmholtz resonator with the L_1 length inertance tube as the resonator and the volume of pulse tube and part of the regenerator as the buffer with line 1–1 as the imagine bottom. At line 1–1, the mass flow rate and pressure wave is in phase. At line 0–0, it can be consider there is an infinite buffer volume. The pressure ratio around line 0–0 is the lowest in the

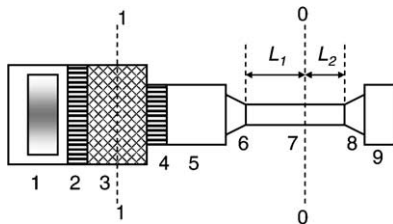


Fig. 16. Inertance tube and Helmholtz resonator. 1. Compressor, 2. cooler, 3. regenerator, 4. cold heat exchanger, 5. pulse tube, 6. tip1, 7. inertance tube, 8. tip2 and 9. buffer.

inertance tube, while the velocity is the maximum in the inertance tube.

In Fig. 16, if the mass flow rate and pressure wave is in phase at line 1–1, the pressure wave must leads a phase angle at the cold end of the pulse tube.

The frequency of the two helmholtz resonator is the frequency of the compressor which is fixed by the design. The length of L_1 and L_2 can be estimated from Ref. [12].

$$L_1 = \frac{\pi}{4} \frac{D^2}{\beta V_P + V_{tip1}} \left(\frac{c}{\omega} \right)^2 \quad (9-1)$$

$$L_2 = \frac{\pi}{4} \frac{D^2}{V_b + V_{tip2}} \left(\frac{c}{\omega} \right)^2 \quad (9-2)$$

Here βV_P means the equivalent volume of the pulse tube. We have chosen β as 3 based on Ref. [13]. The total length of the inertance tube L is equal to L_1 plus L_2 .

Eqs. (9-1) and (9-2) are useful for the explaining of the inertance tube working condition. In Fig. 16, if the diameter of the inertance tube and L_2 are fixed, the decrease of the total length of inertance tube leads to the decrease of L_1 , then the imaginary buffer bottom line 1–1 will move to the left. If the diameter of the inertance tube increased, the length of L_2 is increased, the optimum length of L_1 also will be increased because the imaginary buffer bottom line 1–1 will not be changed so much for the optimum COP.

Eqs. (9-1) and (9-2) are also useful for a rough estimation the inertance tube. This helps us to decrease the numerical calculation job because the calculation speed is not so high even we use the acceleration method. The length of L_1 , L_2 , and L determined by the numerical simulation (makers) and by Eqs. (9-1) and (9-2) (curves) are shown in Figs. 17 and 18. It is shown that Eq. (9-2) has a very good accuracy, while Eq. (9-1) gives much

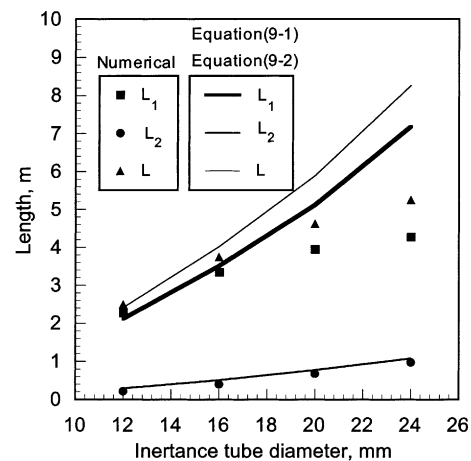


Fig. 17. Inertance tube length and its lowest pressure ratio point vs. inertance tube diameter.

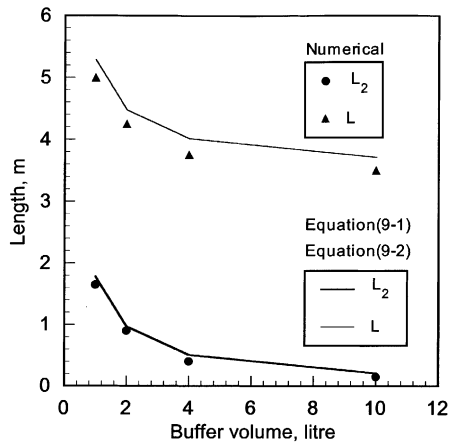


Fig. 18. Inertance tube length and its lowest pressure ratio point vs. buffer volume.

different values from numerical results when the inertance tube diameter is over 16 mm.

The accuracy of Eq. (9-1) may be depend on the value of $(0.25\pi D^2 L_1)/(\beta V_P + V_{ip1})$. With diameter 16 mm, the $(0.25\pi D^2 L_1)/(\beta V_P + V_{ip1})$ by Eq. (9-1) is less than 1.2.

The length of L_1 should be less than one quarter wave length of helium gas of 5 m at 50 Hz and 300 K in order to let the line 1–1 in Fig. 16 in the regenerator. So if the calculation from Eq. (9-1) is over 5 m, it should be chosen less than 5 m.

7. Conclusion

A nodal analytical method based on implicit control volume concept is introduced for inertance tube pulse tube refrigerator. It can simulate the inertance tube pulse tube refrigerator and the thermal acoustic engine. It becomes a general numerical method, which can be used for the simulation of the double inlet pulse tube refrigerator, GM refrigerator, Stirling refrigerator, and other type refrigerators, too. The continuity equation is changed to pressure equation and merged with momen-

tum equation for decreasing the algebraic equations to one group for increasing the calculation speed. A large scale inertance tube pulse tube refrigerator is taken as an example for the calculation.

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